EFFECT OF THE RIGIDITY OF THE STRATA COVERING A WATER-BEARING LAYER ON THE REGULARITY OF ELASTIC FILTRATION CONDITIONS

I. G. Kotov, V. A. Mironenko,
and L. I. Serdyukov

The article discusses the effect of the rigidity of the strata covering a water-bearing level on the laws governing elastic filtration conditions during the process of test evacuations. The problem reduces to solution of the equation for the sag of an infinite slab on an elastic base. It is shown that the rigidity of the covering strata can be neglected only with a sufficiently long duration of the evacuation; corresponding approximate criteria are given.

As is well known, the classical theory of elastic filtration conditions is based on the assumption of the total transmission of a "depression load" to water-bearing rocks. In other words, it is assumed that a decrease in the pressures S at any given point of the covering of the water-bearing stratum brings about an increase in the effective pressure at this point by an amount γS(γ is the volumetric weight of water). There is no doubt with respect to the practical application of this postulation, under the condition that the dimensions of the depression crater considerably exceed the total thickness of the rock coverings, M. However, many filtration problems are solved for conditions when the diameter of the zone of the effect of evacuation, 2R, is commensurate with the value of M, or even less; this occurs, for example, when carrying out relatively short-term evacuations from water-bearing strata, lying at considerable depths. Under such conditions, it is evident that there will be a considerable "hanging" effect of the covering stratum; the pressure on the water-bearing rocks (within the limits of the zone of the evacuation effect) from the side of the covering rocks will be only partially transmitted, in view of the rigidity of the strata and of the limited dimensions of the depression crater. This fact has already been pointed out in a number of earlier published articles [1, 2]. In particular, the authors of [2], proposing an arbitrary law for the transmission of the pressure, note that the bending of the curves of the restoration of the level in their initial segments may be explained precisely by this incomplete transmission of the pressure during the first stages of evacuation.

In distinction from work carried out earlier, in the present article an attempt is made to make a real evaluation of the effect of the "hanging" factor on elastic filtering conditions.

The problem is solved in a simplified statement. Around a borehole of radius r_0 there is set up a depression crater with a radius R; the depressions within the limits of the crater are described by the logarithmic dependence

$$S(r) = S_0 \frac{\ln(R/r)}{\ln(R/r_0)} \tag{1}$$

Here S_0 is the depression in the borehole; r is the instantaneous coordinate.

As a result of the lowering of the hydrostatic pressure, the shelf of covering rock tends to sag, so that the excess effective pressure $p_0 = \gamma S(r)$ is transmitted to the water-bearing rocks from which the vacuation is being carried out.

Leningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 172-176, March-April, 1971. Original article submitted June 19, 1970.

The sagging is prevented by the forces of elastic resistance to the bending of the rocks covering the shelf, and by the forces of elastic resistance from the side of the water-bearing stratum under compression. In view of the limited scales of the deformations of the covering shelf, it is completely admissible to assume that they obey the laws of the theory of elasticity.

In this statement, the problem is reduced to solution of the equation for the axisymmetric sag of an infinite slab on an elastic base [3], which, with application to the conditions under consideration, may be written in the following manner:

$$D\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)\left(l''+\frac{1}{r}l'\right)=p_0(r)-Al(r) \tag{2}$$

$$A=\frac{1+\varepsilon}{am}, \qquad D=\frac{EM^3}{12(1-\nu^2)}$$

Here l is the deformation sagging of the covering rocks, or the compression deformation of the water-bearing rocks; A is the coefficient of proportionality between the compression deformations of the water-bearing rocks and their loading; ε, a, and m are the porosity coefficient, the compressibility coefficient, and the thickness of the water-bearing rocks; E and ν are the Young modulus and the Poisson coefficient (averaged) for the rocks of the covering shelf.

The general solution of this equation has the form [3]

$$l=c_1U_0(\rho)+c_2v_0(\rho)+c_3f_0(\rho)+c_4g_0(\rho)+l_0(\rho) \tag{3}$$

$$\rho'=br, \qquad b=(A/D)^{1/4}, \qquad U_0(\rho)=\mathrm{ber}(\rho)$$

$$v_0(\rho)=-\mathrm{bei}(\rho),\ f_0(\rho)=-2\pi^{-1}\mathrm{kei}(\rho), \qquad g_0(\rho)=-2\pi^{-1}\mathrm{ker}(\rho)$$

Here c_i are integration constants; b is a reduction coefficient; $l_0(\rho)$ is some partial solution of Eq. (2); ker (ρ), kei (ρ), ber (ρ), bei (ρ) are Thomson functions [4].

For the partial case of a force, uniformly distributed around a circle with the reduced radius $\rho = \alpha$, the solution assumes the form [3]

$$l(\rho,\alpha)=\frac{\pi\alpha q}{2Db^3}\left[f_0(\alpha)U_0(\rho)-g_0(\alpha)v_0(\rho)\right] \quad \text{for} \quad \rho\leqslant\alpha \tag{4}$$

$$l(\rho,\alpha)=\frac{\pi\alpha q}{2Db^3}\left[U_0(\alpha)f_0(\rho)-v_0(\alpha)g_0(\rho)\right] \quad \text{for} \quad \rho\geqslant\alpha$$

where q is the intensity of the loading

In accordance with the law adopted in (1) for the distribution of the depression, S, the depression loading (on a circle with the reduced radius $\rho = \alpha$) is characterized by the intensity

$$q(\alpha)=p_0\frac{2\pi r}{2\pi\rho}=\frac{p_0}{b}=\gamma S_0\frac{\ln(\rho_1/\alpha)}{b\ln(\rho_1/\rho_0)}, \qquad \begin{matrix}\rho_0=br_0\\ \rho_1=bR\end{matrix} \tag{5}$$

The total deformation from the whole depression loading can be obtained by integrating expression (4) with respect to α within the limits from ρ_0 to ρ_1, with an intensity, q, corresponding to formula (5)

$$l(\rho)=\frac{\pi\gamma S_0}{2A\ln(R/r_0)}\Big[f_0(\rho)\int\limits_{\rho_0}^{\alpha}\alpha U_0(\alpha)\ln(\rho_1/\alpha)\,d\alpha-g_0(\rho)\int\limits_{\rho_0}^{\alpha}\alpha v_0(\alpha)\ln(\rho_1/\alpha)\,d\alpha$$

$$+U_0(\rho)\int\limits_{\alpha}^{\rho_1}\alpha f_0(\alpha)\ln(\rho_1/\alpha)\,d\alpha-v_0(\rho)\int\limits_{\alpha}^{\rho_1}\alpha g_0(\alpha)\ln(\rho_1/\alpha)\,d\alpha\Big] \tag{6}$$

To solve the integrals in expression (6), we use the relationships

$$\int U_0(x)\,x\,dx=-xv_0'(x), \qquad \int v_0(x)\,x\,dx=xU_0'(x)$$

$$\int f_0(x)\,x\,dx=-xg_0'(x), \qquad \int g_0(x)\,x\,dx=xf_0'(x)$$

$$v_0f_0'+U_0g_0'=U_0'g_0+v_0'f_0+2/\pi x$$

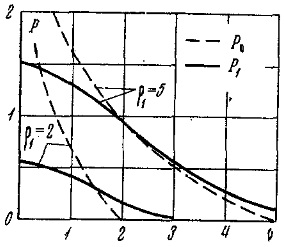

Fig. 1

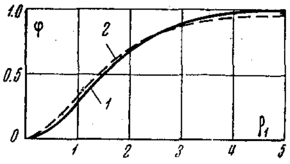

Fig. 2

At $R \gg r_0$ and $r_0 \to 0$, the solution of (6) is brought to the form

$$l(\rho) = \frac{\pi\gamma S_0}{2A\ln(R/r_0)}\left\{2\pi^{-1}\ln\left(\frac{\rho_1}{\rho}\right) + g_0(\rho) - [f_0(\rho_1)v_0(\rho) + g_0(\rho_1)U_0(\rho)]\right\} \quad (7)$$

In particular, the deformation sagging at the center (with $\rho = 0$) is

$$l(0) = \frac{\gamma S_0}{A\ln(R/r_0)}\left[\ln\rho_1 - \frac{\pi}{2}g_0(\rho_1)\right] \quad (8)$$

In view of the fact that the pressure transmitted to the water-bearing stratum is equal to $p_1 = Al$, from formula (8) it follows:

$$p_1(0) = \frac{\gamma S_0}{\ln(R/r_0)}\left[\ln\rho_1 - \frac{\pi}{2}g_0(\rho_1)\right].$$

With an increase in ρ_1, the function $g_0(\rho_1) \to 0$; therefore, complete transmission of the pressure at the point $r = 0$ can take place only at very large values of ρ_1 (when $\ln\rho_1 \ll \ln\rho_0$).

Figure 1 gives curves of the reduced pressures for $\rho_1 = 2$ and $\rho_1 = 5$,

$$P_1(\rho) = p_1(\rho)\frac{\ln(R/r_0)}{\gamma S_0}, \quad P_0(\rho) = p_0(\rho)\frac{\ln(R/r_0)}{\gamma S_0}$$

It is evident from Fig. 1 that, at $\rho_1 = 2$, the actual pressure transmitted to the water-bearing stratum differs sharply from the values of the pressure calculated on the assumption of complete transmission of the depression loading. On the contrary, at $\rho_1 = 5$, the actual and calculated pressures are sufficiently close together within the limits of a considerable part (approximately 85%) of the area of the depression crater.

To evaluate the values of ρ_1 at which the effect of the incomplete transmission of the pressure on the filtration process will be appreciable, we determine the quantity

$$\varphi(\rho_1) = \frac{2\pi}{J}\int_{r_0}^{R} p_1(r)\,r\,dr = \frac{2\pi A}{b^2 J}\int_{\rho_0}^{\rho_1} l(\rho)\,\rho\,d\rho$$

$$J = 2\pi\int_{r_0}^{R} p_0(r)\,r\,dr = \frac{2\pi}{b^2}\int_{\rho_0}^{\rho_1} p_0(\rho)\,\rho\,d\rho$$

Taking account of (7), this quantity can be represented in the form

$$\varphi(\rho_1) = 1 - (2\pi/\rho_1)\,[f_0(\rho_1)\,U_0'(\rho_1) - g_0(\rho_1)\,v_0'(\rho_1) - f_0'(\rho_1)]$$

The quantity $\varphi(\rho_1)$ obviously corresponds to the ratio of the volume of the elastic reservoirs being evacuated, i.e., the actual and the calculated, with the usual assumption with respect to the complete transmission of the depression loading. If it is assumed that the conductivity of the stratum is invariable, then

$$\varphi(\rho_1) = a_0^*/a^* = \mu^*/\mu_0^*$$

where a_0^* and μ_0^* are the coefficient of piezoconductivity and the yield of water, determined without taking account of the "hanging" factor; a^* and μ^* are the same, but taking account of the "hanging" factor. The dependence $\varphi(\rho_1)$ is shown in Fig. 2 (Curve 1).

Thus, the effect of the "hanging" factor comes down, in the final analysis, to the fact that the coefficient of piezoconductivity is found to be a quantity depending on the dimensions of the zone of the evacuation effect, and varying from $k(1+\varepsilon)/\gamma E_1\varepsilon$ (E_1 is the elastic modulus of water) to its limiting value a_0^* with a sufficiently long duration of the evacuation.

On the other hand, it can be shown that, with single evacuations, the zone from which more than 85% of the elastic reserves of water come, is limited by the calculated radius

$$R(t) = 2\sqrt{a_0^* t} \tag{9}$$

and if, in this case, the depression curve is approximated by the logarithmic law (1), the volumes of the elastic reserves actually evacuated after a time t are found to be equal to the volume of a depression crater with the above calculated radius.

Consequently, it may be postulated that, for the given problem, in all the above reduced relationships, the quantity R is understood as a calculating parameter, determined by formula (9); therefore, the quantity $\rho_1^2 = b^2R^2$ is found to be directly proportional to the time t, so that the function $\varphi(\rho_1)$ (Fig. 2) can be equated to some function of the time $f(t)$; $f(t) \equiv \varphi(2b\sqrt{a_0^* t})$.

Thus, for the conditions of the problem under consideration, it is possible to speak of the change in the calculated value of the elastic water removal with time.

In view of this, it is apropos to draw an analogy between the problem under consideration and the problem of filtration in water-bearing strata without pressure. As is well knwon, with relative short-term evacuations, the calculated water removal, μ, from a pressureless water-bearing stratum is found to be a quantity which depends essentially on the time; this dependence can be written in the form [5]

$$\varphi_1 = \frac{\mu}{\mu_0} = \frac{A_1B_1t - (1 - e^{-A_1B_1t})}{A_1B_1t + A_1^{-1}(1 - e^{-A_1B_1t})} \qquad (A_1, B_1 = \text{const}) \tag{10}$$

Here t is the time; μ_0 is the value of the water removal at $t \to \infty$. Taking this circumstance into account, we shall attempt to approximate the curve in Fig. 2 by a formula of the type

$$\varphi = \frac{A_2\rho_1^2 - (1 - e^{-A_2\rho_1^2})}{A_2\rho_1^2 - B_2(1 - e^{-A_2\rho_1^2})} \tag{11}$$

At $A_2 = 0.4$ and $B_2 = 0.6$, the approximation is found to be sufficiently satisfactory (Fig. 2, curve 2). From a comparison of (11) and (10) it follows that the case of filtration under consideration may, with a certain amount of approximation, be approximated by relationships derived for pressureless filtration with variable water removal [5]; in this case, the quantity B_1t in (10) corresponds to the quantity ρ_1^2 in (11). While in the case of pressureless filtration the change of the water removal with time may be neglected [5] only with $B_1t > 5$-10, in the problem under consideration, the corresponding criterion has the form $\rho_1^2 > 5$-10* or

$$\frac{R}{M} > (1.2 \div 1.7)\left[\frac{m}{M}\,\frac{aE}{(1+\varepsilon)(1-\nu^2)}\right]^{1/4} \tag{12}$$

Starting from expression (9), we can obtain a corresponding time criterion for a single evacuation, with a constant output

$$\frac{a_0^* t}{M^2} > (0.4 - 0.7)\left[\frac{m}{M}\,\frac{aE}{(1+\varepsilon)(1-\nu^2)}\right]^{1/2} \tag{13}$$

Consequently, all the calculating formulas for elastic filtration conditions may be used for the analysis of test evacuations only when they are of sufficient duration. This fact, in particular, considerably restricts the applicability of a number of express methods for study of the filtration parameters for sufficiently deep-lying water-bearing strata.

Finally, the solution presented here describes the process of the transmission of pressure with a certain degree of approximation. In particular, it was assumed that the shelf of covering rock sags in accordance with the theory of thin slabs which, generally speaking, is admissible [6] only with sufficiently large values of ρ_1. Furthermore, definite errors in the criterion (13) are introduced by the assumption that the

*Since the difference between the values of P_1 and P_0 changes sign with an increase of ρ, at $\rho_1^2 = 5$, the calculated water removal already differs from the actual value by less than 25% (Fig. 2).

quantity R varies in accordance with the law (9). Finally, it is important to bear in mind that, in actual fact, the shelf of covering rock will sag, not in a manner similar to a solid slab, but as a stratified system; under these circumstances, as a result of the presence of undrained pressure strata in this shelf, there will be sagging of the water-bearing layer covering the water-bearing stratum being tested; this takes place under the action of the difference in heads arising during evacuation.

All these factors are evidence of the fact that the formulas obtained above are constructed for considerably simplified conditions. It must, however, be noted that, at the present-day level of the investigations, a more accurate statement of the problems under consideration would have no practical meaning since, for this, in each actual stratum, before carrying out the evacuation there would be required reliable data on the structure of the covering shelves and on the strength parameters of the rocks of which they are made up. Therefore, here only a more modest problem has been posed: to make an evaluation, even if only approximate, of the lower limit of the applicability of the generally accepted equations of elastic filtration conditions.

To evaluate the real quantities which, in practice, characterize the criteria (12) and (13), as an example, we shall consider a water-bearing stratum, the Buchak in the Yuzhno-Belozersk iron-ore deposit. According to the results of investigations in the VNIMI [All-Union Scientific-Research Institute for Mine Surveying], it is characterized by the following parameters: $E \approx 0.5 \cdot 10^4$ kg/cm^2; $a = 7 \cdot 10^{-4}$ cm^2/kg; $\varepsilon = 0.7$; $\nu = 0.4$; m = 15 m. Since M = 250 m, from (12) we obtain

$$\frac{R}{M} > (1.2 - 1.7)\left[\frac{15 \cdot 0.5 \cdot 10^4 \cdot 7 \cdot 10^{-4}}{250 \cdot 1.7 \cdot (1 - 0.16)}\right]^{1/4} \approx 0.75 \text{ to } 1.1$$

Consequently, in order that the results of a single evacuation, carried out from the Buchak stratum, may be analyzed without taking the "hanging" factor into account, it is necessary, as a minimum, that the radius of the depression crater exceed the thickness of the covering shelves.

If it is assumed that the ratio $a/(1+\varepsilon)$ is of the same order of magnitude as the quantity E^{-1} (this assumption, as a rule will decrease the role of the "hanging" factor with evacuation from sandy water-bearing strata), and that $\nu = 0.3$–0.4, we obtain the approximate extimates

$$\frac{R}{M} > (1.2 - 1.7)(m/M)^{1/4}, \qquad \frac{a_0{}^* t}{M^2} > (0.40 - 0.70)(m/M)^{1/2} \tag{14}$$

The authors thank S. G. Lekhnitskii and V. M. Shestak for their useful observations.

LITERATURE CITED

1. V. M. Shestakov, "Problems in the theory of elastic filtration conditions" Zh. Prikl. Mekhan. i Tekh. Fiz., No. 3 (1962).
2. E. F. Afanas'ev and V. N. Nikolaevskii, "Nonlocal elastic filtration and reduced pressure conditions in deep strata," Zh. Prikl. Mekhan. i Tekh. Fiz., No. 5 (1969).
3. B. G. Korenev, Problems in the Theory of Elasticity and Thermal Conductivity Solved by Bessel Functions [in Russian], Izd. Fizmatgiz, Moscow (1960).
4. G. B. Dwight, Tables of Integrals and Other Mathematical Formulas [Russian translation], Izd. Inostr. Lit. (1948).
5. V. A. Mironenko and L. I. Serdyukov, "Application of the method of integral ratios in analysis of the results of test evacuation in stratified shelves," Izv. VUZ. Geologiya i Razvedka, No. 11 (1969).
6. S. P. Timoshenko, The Stability of Elastic Systems [in Russian], Izd. Gostekhizdat, Moscow (1946).